数理情報工学演習第二

Matrix scaling and its applications

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Let $\mathcal{B}$ be the set of matrices $\boldsymbol{A} \in \mathbb{R}^{p \times p}$ satisfying $\boldsymbol{A}\boldsymbol{e} = \boldsymbol{e}$ and $\boldsymbol{e}^\top \boldsymbol{A} = \boldsymbol{e}^\top$, where $\boldsymbol{e} = (1, \ldots, 1)^\top \in \mathbb{R}^p$.

Theorem 1 (Marshall and Olkin [3])**.** Let $\boldsymbol{A}$ be a positive definite matrix. Then there exists a unique positive diagonal matrix $\boldsymbol{D}$ such that $\boldsymbol{D}\boldsymbol{A}\boldsymbol{D} \in \mathcal{B}$.

The same conclusion as Theorem 1 holds under weaker conditions. Refer to [3] for details. An application of Theorem 1 to general indices is discussed in [5].

Theorem 2 (Sinkhorn [6])**.** Let $\boldsymbol{A}$ be a positive square matrix. Then there exist positive diagonal matrices $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ such that $\boldsymbol{D}_1\boldsymbol{A}\boldsymbol{D}_2 \in \mathcal{B}$. The matrices $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are unique up to a scalar factor.

The same conclusion as Theorem 2 holds under weaker conditions. Refer to [7] for details. Applications of Theorem 2 to rating and input-output analysis are discussed in, for example, [1] and [4], respectively. See also [2] for further information.

References

[1] Langville, A.N. and C.D. Meyer (2012) *Who's #1?: The Science of Rating and Ranking*, Princeton University Press. （日本語訳：岩野和生・中村英史・清水咲里,「レイティング・ランキングの数理：No.1 は誰か？」, 共立出版）

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[2] Idel, M. (2016). A review of matrix scaling and Sinkhorn's normal form for matrices and positive maps, Preprint arxiv:1609.06349.

[3] Marshall, A. W. and Olkin, L. (1968). Scaling of matrices to achieve specified row and column sums, *Numer. Math.*, **12**, 83–90.

[4] 森岡涼子・津田宏治 (2014). 情報幾何的分解に基づく地方産業連関表の将来推計, 数理解析研究所講究録, 85–102.

[5] Sei, T. (2016). An objective general index for multivariate ordered data, *J. Multivariate Anal.*, **147**, 247–264.

[6] Sinkhorn, R. (1964). A relationship between arbitrary positive matrices and doubly stochastic matrices, *Ann. Math. Statist.*, **35**, 876–879.

[7] Sinkhorn, R. and Knopp, P. (1967). Concerning nonnegative matrices and doubly stochastic matrices, *Pacific J. Math.*, **21**, 343–348.

レポート課題

以下の要領で提出してください。

提出先 計数教務室

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注意 氏名，学生証番号，科目名を明記してください。レポートは日本語で良いです。

課題 Answer to at least one of the following questions.

1. Prove Theorem 1.
2. Prove Theorem 2.
3. Describe an application of Theorem 1.
4. Describe an application of Theorem 2.
5. Produce a balancing toy presented in the lecture. Attach a photograph of it.